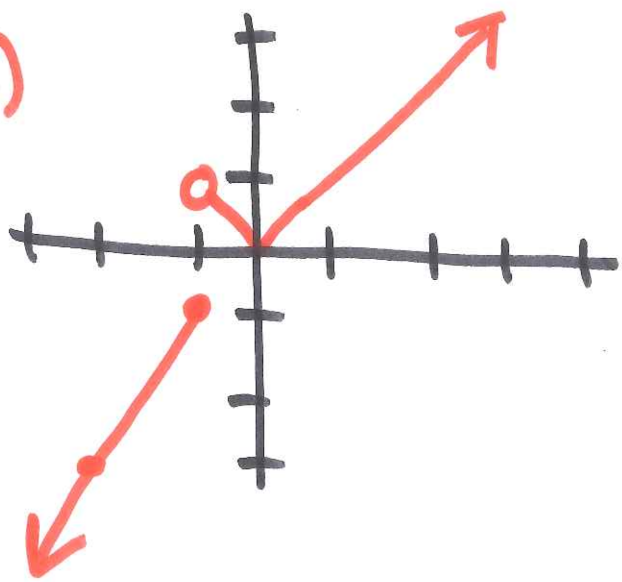
Chapter 4 - Day 2

<u>Ex</u>: Consider the graph of

$$g(x) = \begin{cases} 2x+1 & \text{if } x \le -1 \\ |x| & \text{if } x > -1 \end{cases}$$

Where is $g'(x)$ undefined?

g(x)



$\boxed{x = 0, -1}$

Is $g(x)$ continuous at those points?

not continuous at $x=-1$

continuous at $x=0$

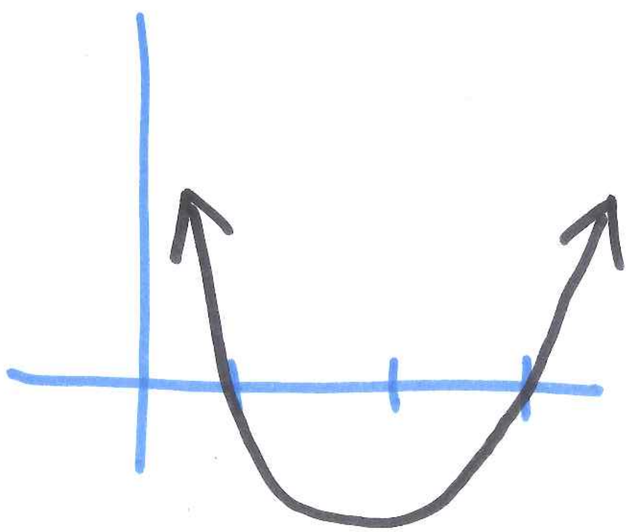
<u>Ex</u>: Consider

$$f(x) = |x^2 - 4x + 3|$$

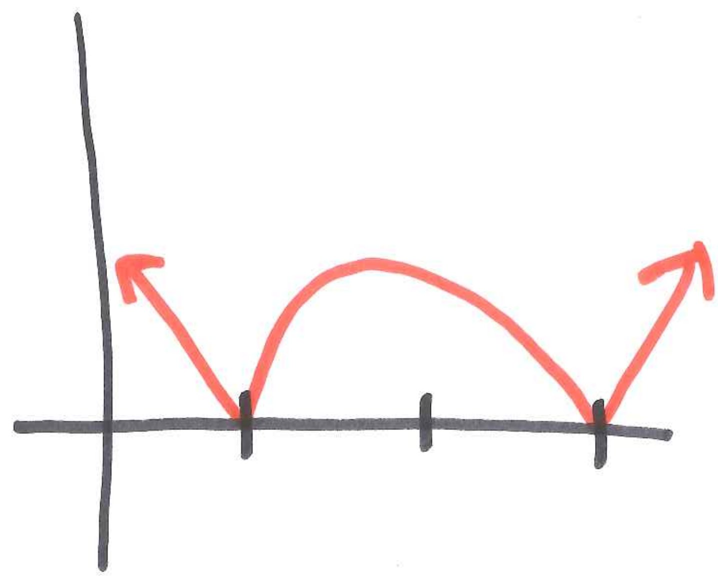
* think about

$y = x^2 - 4x + 3$

$y = (x-1)(x-3)$



now take the absolute value



a) Where is $f'(x)$ undefined?

at $x = 1, 3$

b) Is $f(x)$ continuous at those points?

Yes - continuous at both $x = 1, 3$

Ex: let $f(x) = (x+4)^2$

a) find A, B, C such that

$$\frac{f(x+h)-f(x)}{h} = Ax + Bh + C$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h+4)^2-(x+4)^2}{h}$$

$$= \frac{(\cancel{x^2}+2xh+\cancel{8x}+h^2+8h+\cancel{16})-(\cancel{x^2}+\cancel{8x}+\cancel{16})}{h}$$

$$= \frac{2xh+h^2+8h}{h} = 2x+h+8$$

* play the matching game!

$$\boxed{A=2,\ B=1,\ C=8}$$

b) Show $f'(x) = 2x + 8 = 2(x+4)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} 2x + h + 8$$

$$= 2x + 0 + 8$$

$$= 2x + 8 = 2(x+4) \checkmark$$

c) Find $f'(5)$ and write the equation of the tangent line to the graph of f at $x=5$.

$f'(5) = 2(5) + 8 = 18$ "slope"

$f(5) = (5+4)^2 = 81$ point $(5, 81)$

tangent line $y - 81 = 18(x - 5)$

$$y - 81 = 18x - 90$$

$$\boxed{y = 18x - 9}$$

Ex: let $g(x) = \dfrac{1}{x+2}$

a) find $g'(x)$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \right] \frac{(x+h+2)(x+2)}{(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{(\cancel{x}+\cancel{2}) - (\cancel{x}+h+\cancel{2})}{h(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{-h}{h(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h+2)(x+2)}$$

$$= \frac{-1}{(x+2)^2} = -(x+2)^{-2}$$

b) find $g'(3)$

$$g'(3) = \frac{-1}{(3+2)^2} = \frac{-1}{5^2} = \frac{-1}{25}$$